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COMBINED NATURAL AND FORCED CONVECTION HEAT TRANSFER FROM HORIZONTAL CYLINDERS TO WATER

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Abstract—Experimental data for convective heat transfer from horizontal cylinders (diameter approximately $\frac{1}{2}$ in.) to water in crossflow have been obtained. It is shown that these data can be relegated to four zones, depending upon the value of the ratio $G_f R_f^{-2}$, where G_f and R_f are the Grashof and Reynolds numbers, respectively, with properties evaluated at the mean film temperature. In the first zone, defined by $G_f R_f^{-2} < 0{\cdot}5$, the predominant heat transfer mechanism is forced convection, and previously published forced convection equations may be applied with negligible error (less than about 5 per cent). In the second zone, defined by $0{\cdot}5 < G_f R_f^{-2} < 2$, the predominant heat transfer mechanism is still forced convection, but here natural convection affects the overall heat transfer coefficient by as much as 10 per cent and this is not considered to be negligible. Empirical correlation equations are presented by means of which it is possible to account for natural convection in zone two for three directions of forced flow: vertically upward, horizontal, and vertically downward. Zone three, defined by $2 < G_f R_f^{-2} < 40$, is a region in which natural and forced convection effects are of the same order of magnitude. In a subregion of this zone ($4 < G_f R_f^{-2} < 40$) the heat transfer process is unsteady when the direction of the forced flow is horizontal; under such conditions the heat transfer coefficient oscillates with random period between two extreme values. It is hypothesized that this oscillation is a result of periodic changes in the boundary layer flow—the lower extreme value of the heat transfer coefficient is believed to correspond to the periodic establishment of laminar boundary layer flow resembling the flow associated with pure natural convection, and the higher extreme value is believed to correspond to the periodic establishment of the kind of separated boundary layer flow normally encountered near cylinders in forced crossflow. Zone four, defined by $G_f R_f^{-2} > 40$, is a region in which natural convection effects predominate. A simple theory has been developed to describe the behavior of the heat transfer coefficient in zone four. A fifth zone is identified which contains none of the data gathered here.

NOMENCLATURE

A, a, constants;
B, constant;
C, c, constants;
C_p, specific heat at constant pressure;
D, cylinder diameter, constant;
E, error, per cent:

$$E = \left(\frac{N_{\text{calc}} - N_{\text{exp}}}{N_{\text{exp}}}\right) 100;$$

E_{av}, average error, per cent:

$$E_{av} = \frac{1}{n}\sum_{i=1}^{n} E_i;$$

E_m, mean deviation, per cent:

$$E_m = \frac{1}{n}\sum_{i=1}^{n} |E_i|;$$

E_{max}, maximum deviation, per cent;
G, Grashof number: $G = g\beta(\Delta t)\, D^3 \nu^{-2}$;
g, gravitational acceleration, constant;
h, heat transfer coefficient;
K, constant;
k, thermal conductivity;
m, constant;
N, Nusselt number: $N = hDk^{-1}$;
n, constant;
P, Prandtl number: $P = C_p \mu k^{-1}$;
q, rate of heat transfer per unit area;
R, Reynolds number: $R = DU\nu^{-1}$;
r, constant;
T, absolute temperature;
t, temperature;
U, velocity;
x, distance, constant.

Greek

- β, coefficient of volumetric expansion;
- Δt, temperature difference: $\Delta t = (t_s - t_b)$;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- θ, angle;
- σ, sample standard deviation of error, per cent:

$$\sigma = \left[\frac{1}{n-1}\left\{\sum_{i=1}^{n} E_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} E_i\right)^2\right\}\right]^{\frac{1}{2}}$$

Subscripts

- b, refers to bulk fluid conditions far from a heat transfer surface;
- F, refers to forced convection;
- f, refers to fluid conditions at the mean film temperature;
- N, refers to natural convection;
- s, refers to fluid conditions at a heat transfer surface.

INTRODUCTION AND STATEMENT OF OBJECTIVE

WHEN a surface is in contact with a fluid whose bulk temperature is different from the temperature of the surface, a nonuniform temperature field is established in the fluid. This temperature field causes changes in density which may, via buoyant forces, cause the fluid to move relative to the surface. When heat is transferred under such conditions, the phenomenon is called "natural convection heat transfer." When motion is induced in the fluid by other means, such as by means of a pump or fan, the associated process is called "forced convection heat transfer." In all forced convection situations the mechanism of natural convection is still operative, since density gradients and associated buoyant force fields still exist. Thus, strictly speaking, natural convection effects are present in all convective heat transfer processes.

The Grashof number, $G = g\beta(\Delta t)\ D^3\nu^{-2}$, which represents the ratio of buoyant to viscous forces, and the Reynolds number, $R = UD\nu^{-1}$, which represents the ratio of inertia to viscous forces, play key roles in the evaluation of heat transfer by natural and forced convection. Since heat transfer coefficients generally increase with increasing G and R, it follows that forced convection effects can be made either large or small compared to natural convection effects, by appropriate selection of values of D, U and Δt. In particular, both R and G approach zero as D approaches zero (other variables being constant); but in this case G approaches zero faster than R, since D is raised to the third power in G and to the first power in R. Thus, heat transfer due to natural convection can be made small compared to forced convection by either utilizing high velocities, or, in case of moderate velocities, by choosing a body having a small characteristic dimension. Many investigators of convective heat transfer have chosen one or the other of these situations, thereby enabling them to consider natural convection effects to be negligible compared to forced convection. In such instances, the heat transfer is said to occur by forced convection alone. Relatively little work has been done in the region where both natural and forced convection effects are of comparable magnitude.

The objective of this investigation was to study convective heat transfer under conditions where natural and forced convection effects are of the same order of magnitude. More specifically, this paper presents a study of combined natural and forced convection heat transfer from a horizontal circular cylinder to crossflows of water flowing in the vertically upward, vertically downward and horizontal directions.

LITERATURE SURVEY

Since there is no sharp dividing line between natural and forced convection, it is helpful to define a criterion whereby either forced or natural convection effects may be neglected relative to the other. It is well established in the literature that the appropriate criterion to use in order to determine when natural convection effects may be neglected in the presence of forced flows is the ratio GR^{-n}, where n is a number between approximately 2 and 3. Kreith [1] has made an order of magnitude study of the boundary layer equation for one-dimensional flow and concluded that when $GR^{-2} > 1$ natural convection effects cannot be ignored in forced flows. Sparrow and Gregg [2] have analyzed the flow over vertical flat plates and have arrived at the conclusion that if an accuracy of 5 per cent is sufficient in computation of overall heat transfer rates, then the forced convection heat transfer result can be used whenever $G_f R_f^{-2} < 0{\cdot}255$ for $0{\cdot}01 < P_f < 10$.

Sharma and Sukhatme [3] studied heat transfer from heated cylinders to air in crossflow. They used cylinders having diameters equal to 1·25, 2·50 and 5·05 cm and arrived at the conclusion that as the velocity of flow is increased, natural convection is the dominant mode of heat transfer until a "first point of transition" is reached where $G_f R_f^{-3{\cdot}25} = 0{\cdot}185 \pm 0{\cdot}010$. As the velocity is further increased, the heat transfer process enters a transition zone. Eventually, a "second point of transition" is reached, defined by $G_f R_f^{-1{\cdot}8} = 0{\cdot}58 \pm 0{\cdot}13$, beyond which the rate of heat transfer can be closely approximated by forced convection equations.

Oosthuizen and Madan [4] have investigated heat transfer from horizontal cylinders (0·75 in. $< D <$ 1·50 in.) to air flowing in a vertically upward direction (here $\theta = 0$, where θ is the angle between the direction of the forced flow and the vertical). Their conclusion was that for combined natural and forced convection:

$$N_f = \left[1 + 0{\cdot}18\left(\frac{G_f}{R_f^2}\right) - 0{\cdot}011\left(\frac{G_f}{R_f^2}\right)^2\right] N_{f,F}, \quad (1)$$

where $N_{f,F}$ is calculated from a forced convection equation having the form:

$$N_{f,F} = 0{\cdot}464 R_f^{0{\cdot}5} + 0{\cdot}0004 R_f. \quad (2)$$

These workers further concluded that in the region defined by $G_f R_f^{-2} < 0{\cdot}28$, N_f differs from $N_{f,F}$ by no more than 5 per cent. Oosthuizen and Madan [5] later expanded the scope of their initial study to include cases wherein the angle θ assumed the values 90, 135 and 180 deg in addition to $\theta = 0$. They concluded that nearly pure forced flow exists when

$$G_f R_f^{-2} < 0{\cdot}10 \text{ for } \theta = 0 \text{ deg},$$
$$G_f R_f^{-2} < 0{\cdot}53 \text{ for } \theta = 90 \text{ deg},$$
$$G_f R_f^{-2} < 0{\cdot}04 \text{ for } \theta = 135 \text{ deg},$$
$$G_f R_f^{-2} < 0{\cdot}01 \text{ for } \theta = 180 \text{ deg}.$$

A number of investigators have attempted to develop correlation equations for combined natural and forced convection by the vectorial addition of natural and forced convection effects. The first to make this attempt was Van der Hegge Zijnen [6]. His procedure was to first develop two equations for natural and forced convection which, for air, are

$$N_{f,N} = 0{\cdot}35 + 0{\cdot}24 G_f^{1/8} + 0{\cdot}41 G_f^{1/4}, \quad (3)$$

$$N_{f,F} = 0{\cdot}35 + 0{\cdot}50 R_f^{1/2} + 0{\cdot}001 R_f. \quad (4)$$

Then, based on the hypothesis that "the total heat transfer is the vector sum of natural and forced heat transfer" he derived, using equations (3) and (4), the following relation in which the Nusselt number for combined flow, N_f, appears implicitly:

$$(N_f - 0{\cdot}35)^2 = (0{\cdot}24 G_f^{1/8} + 0{\cdot}41 G^{1/4})^2 + (0{\cdot}5 R_f^{1/2} + 0{\cdot}001 R_f)^2. \quad (5)$$

Van der Hegge Zijnen conducted experiments with heated cylinders ($0{\cdot}01 < D < 0{\cdot}904$ cm) in air and found that equation (5) gave fairly good correlation for velocities greater than 20 cm/s; for velocities less than 20 cm/s the equation predicts values for N_f that are too high. Van der Hegge Zijnen's approach is difficult to justify on physical grounds since it involves the "vectorial addition" of Nusselt numbers, which are not vectors.

Hatton, James and Swire [7] have suggested a vectorial addition method for correlating combined natural and forced convection heat transfer which is based upon the observation that, if the work done by the buoyant force on a fluid element is equated to its gain in kinetic energy, then it follows that

$$R = (2G)^{1/2}. \quad (6)$$

The quantity R in equation (6) may be regarded as an "equivalent Reynolds number for natural convection." The principle of equivalence embodied in equation (6), together with the assumption that the same form of equation represents both natural and forced convection, namely,

$$N_N(T_f/T_\infty)^{-m} = A + B(GP)^a, \qquad (7)$$

$$N_F(T_f/T_\infty)^{-m} = C + D(R)^c, \qquad (8)$$

enabled Hatton *et al.* to correlate experimental data on combined natural and forced convection heat transfer from horizontal cylinders to air which they had obtained in the range of Reynolds numbers from 0·01 to 45 for $\theta = 0$, 90 and 180 deg. The correlation procedure involves the replacement of R and G in equations (7) and (8) by their equivalents in terms of G and R, respectively. However, since the natural convection Reynolds number is based upon some characteristic velocity in the fluid, while the forced flow Reynolds number is based upon the free stream velocity, Hatton *et al.* had to modify the relationship $R = (2G)^{\frac{1}{2}}$ derived for a *fluid element* before R and G in equations (7) and (8) could be replaced by their equivalents. They found that if A is *forced* to equal $C = 0{\cdot}384$ and B, D, a, c and m are taken to be 0·59, 0·581, 0·184, 0·439 and 0·154, respectively, then the same Nusselt number is obtained from equations (7) and (8) when

$$R = 1{\cdot}03(GP)^{0{\cdot}418}. \qquad (9)$$

By adding the forced and natural convection Reynolds numbers to obtain an "effective Reynolds number," R_{eff}, Hatton *et al.* found they could correlate their combined natural and forced convection data by the following equation:

$$N_f(T_f/T_\infty)^{-0{\cdot}154} = 0{\cdot}384 + 0{\cdot}581R_{\text{eff}}^{0{\cdot}439} \qquad (10)$$

where

$$R_{\text{eff}}^2 = R_f^2\left[1 + 2{\cdot}06\cos\theta\frac{(G_fP_f)^{0{\cdot}418}}{R_f} + 1{\cdot}06\frac{(G_fP_f)^{0{\cdot}836}}{R_f^2}\right].$$

Jackson and Yen [8] have adopted the same basic vectorial addition method employed by Hatton *et al.* and have demonstrated that this method can be used to correlate the data of Oosthuizen and Madan [4].

The method originated by Hatton *et al.* and later applied by Jackson and Yen correlate the data to which they were applied reasonably well. However, there is a step in the procedure that is unsatisfactory from the phenomenological point of view. This step consists of *forcing* the natural or forced convection equations to satisfy certain conditions that are contrary to the dictates of experimental results. For example, Hatton *et al.* found that computer solutions for the optimum values of the constants in equations (7) and (8) based on the data yielded:

$$N_{f,N}(T_f/T_\infty)^{-0{\cdot}154} = 0{\cdot}525 + 0{\cdot}422(G_fP_f)^{0{\cdot}315}, \qquad (11)$$

$$N_{f,F}(T_f/T_\infty)^{-0{\cdot}154} = 0{\cdot}384 + 0{\cdot}581(R_f)^{0{\cdot}439}. \qquad (12)$$

However, in their vectorial addition procedure the constant 0·525 in equation (11) was arbitrarily changed to equal the value 0·384 as in equation (12) and, with this change, another correlation for natural convection was found, namely,

$$N_{f,N}(T_f/T_\infty)^{-0{\cdot}154} = 0{\cdot}384 + 0{\cdot}59(G_fP_f)^{0{\cdot}184}. \qquad (13)$$

Although equation (13) represents the natural convection data reasonably well, the procedure used to obtain this correlation imposes an artificial constraint upon the data. As a result of this constraint, the functional dependence of $N_{f,N}$ on (G_fP_f) is not realistically represented by equation (13). Jackson and Yen [8] imposed a similar artificiality with respect to their forced convection equation.

Gebhart and Pera [9] have reported the results of an experimental study of heat transfer from fine horizontal wires of various lengths to fluids having various Prandtl numbers. Heat transfer characteristics were determined for the entire spectrum of processes from natural, through combined, to forced convection. They concluded that it is reasonable to delineate the two boundaries of combined convection (predominantly forced and predominantly natural) by $C = GR^{-n}$ where C and n are determined from experimental data (n is between 2 and 3).

EXPERIMENTAL PROCEDURE AND DATA

The experimental portion of the present investigation was conducted by placing a cylindrical, electrically heated test specimen horizontally in a water tunnel, such that the flow of water was normal to the axis of the test specimen (crossflow). Three directions of forced flow were investigated: vertically upward, horizontal and vertically downward. Simultaneous measurements of velocity, power input to the test specimen, surface temperature and bulk water temperature were made, from which the heat transfer coefficient and all other pertinent parameters could be calculated. The ranges of the experimental variables were:

q: 10302–77046 Btu/hft^2,

U: 0–0·43 ft/s,

t_b: 50, 70, 100, 140 deg F,

Δt: 23·4–163·3 deg F.

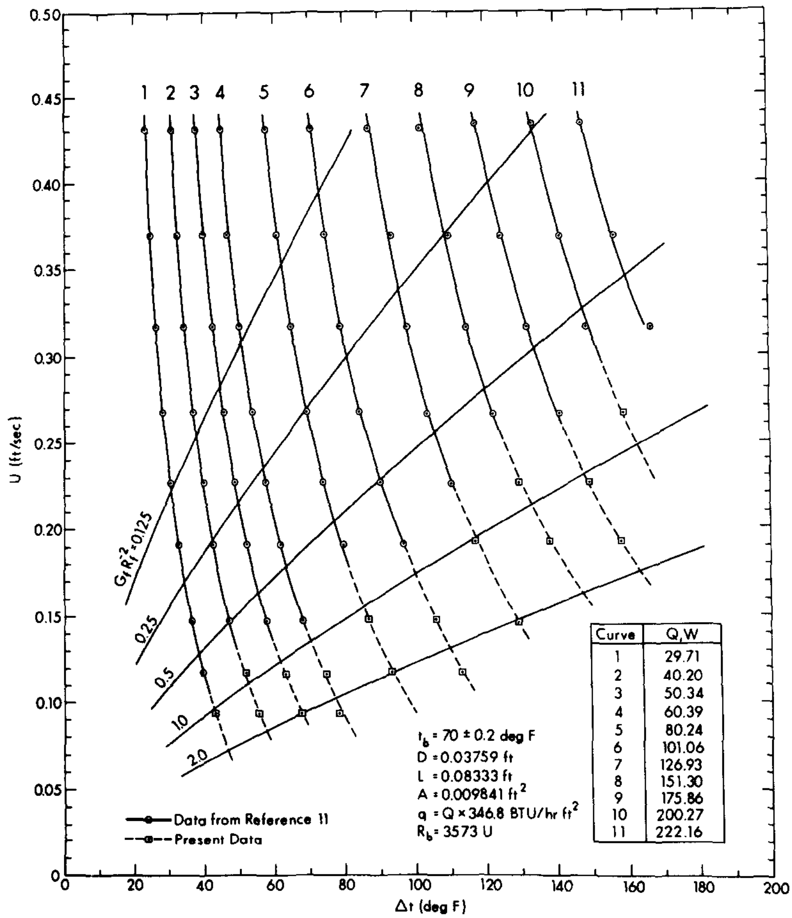

FIG. 1. Experimental data obtained using test cylinder with D = 0·4511 in. and horizontal forced flow (θ = 90 deg).

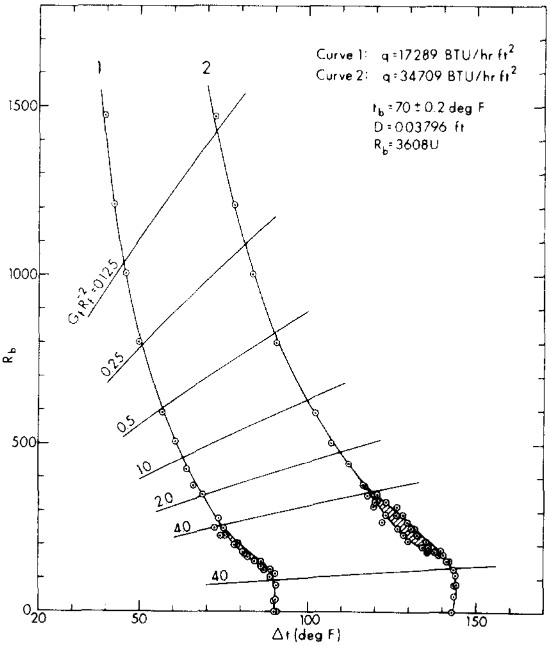

FIG. 2. Experimental data obtained using test cylinder with $D = 0{\cdot}4555$ in., $t_b = 70$ deg F and horizontal forced flow ($\theta = 90$ deg).

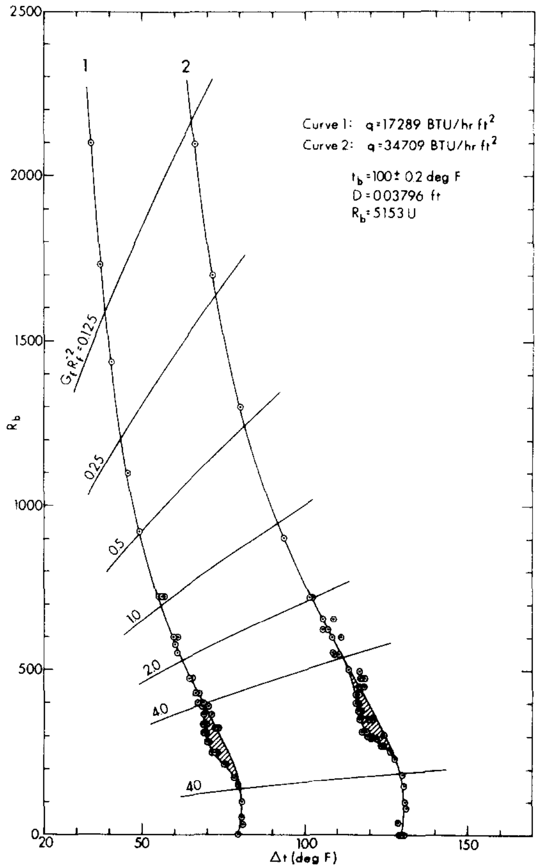

FIG. 3. Experimental data obtained using test cylinder with $D = 0{\cdot}4555$ in., $t_b = 100$ deg F and horizontal forced flow ($\theta = 90$ deg).

The water tunnel used here is described in detail in [10]. This tunnel consists basically of a closed loop through which water can be pumped at controlled and measured values of velocity, temperature and pressure. For purposes of the present investigation the tunnel was disassembled and rearranged so as to provide flows in the vertically upward or horizontal or vertically downward directions ($\theta = 0$, 90 or 180 deg, respectively). The test specimens and experimental test procedures followed in the present investigation are identical to those described in [11].

For convenience of discussion, the experiments performed during this investigation can be divided into two sets. The first set was obtained with the same test specimen ($D = 0{\cdot}4511$ in.) and for the same bulk temperatures (50, 70, 100 and 140 deg F) and for the same direction of forced flow (horizontal: $\theta = 90$ deg) that were used in [11]. However, all the data in [11] correspond to values of $G_f R_f^{-2}$ less than 0·5, whereas the present set covers the range of values of $G_f R_f^{-2}$ from 0·5 to 2. Thus, the present data represent a four-fold extension of the data obtained in [11] with respect to the value of $G_f R_f^{-2}$. The experimental data points of set one for $t_b = 70$ deg F were plotted in the manner shown, by way of example, in Fig. 1; similar plots for t_b equal to 50, 100 and 140 deg F were made which are not reproduced here.

The data of set two were obtained using a different test specimen* ($D = 0{\cdot}4555$ in.); these

* The test specimen used to obtain the data in set one developed an interval short circuit and had to be discarded.

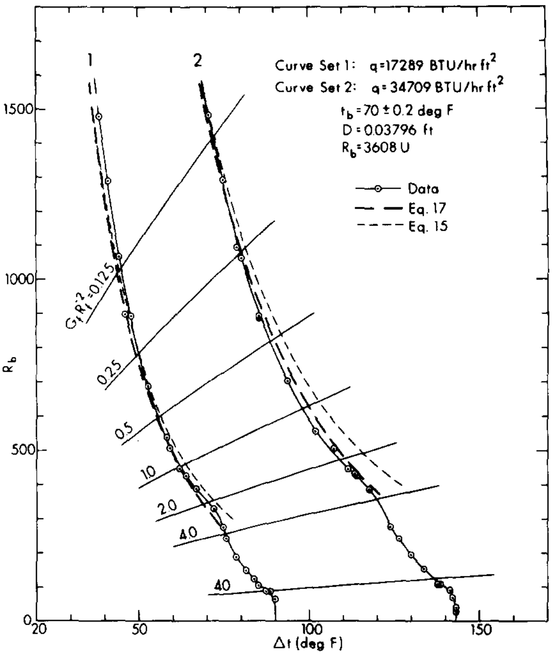

FIG. 4. Experimental data obtained using test cylinder with $D = 0{\cdot}4555$ in., $t_b = 70$ deg F and vertically upward forced flow ($\theta = 0$ deg); plots of equations (15) and (17).

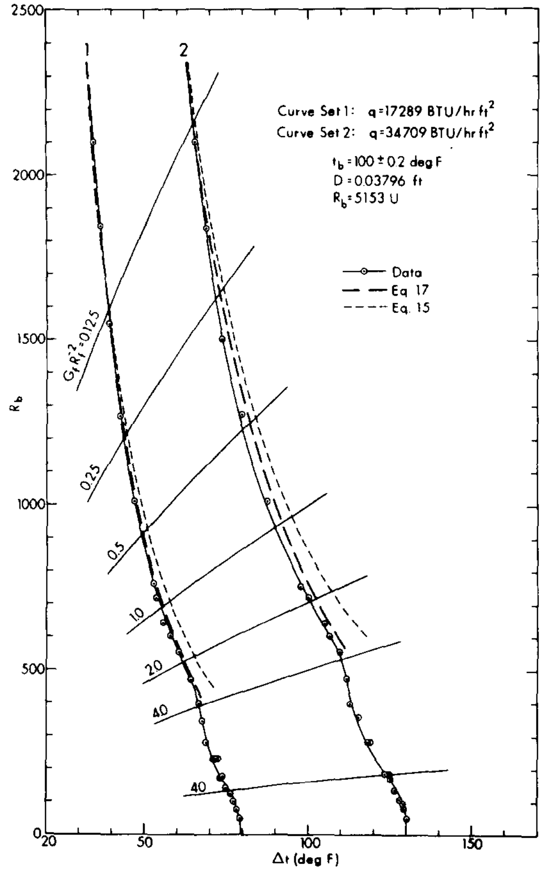

FIG. 5. Experimental data obtained using test cylinder with $D = 0{\cdot}4555$ in., $t_b = 100$ deg F and vertically upward forced flow ($\theta = 0$ deg); plots of equations (15) and (17).

tests provided the data for $\theta = 0$, 90 and 180 deg indicated in Figs. 2–6. For purposes of analysis, curves were fared through the points of both sets of data,* and the intersections of these curves with the family of lines of constant $G_f R_f^{-2}$ shown in the figures were recorded as indicated in Tables 1 and 2. The entries in Tables 1 and 2 were used to develop the correlation equations presented below.

The data for horizontal forced flow in Figs. 2 and 3, and to a lesser extent for vertically upward flow in Figs. 4 and 5, exhibit a kind of anomalous behavior in the zone defined by $2 < G_f R_f^{-2} < 40$ which requires explanation. For horizontal flow the surface temperature of the test specimen was found to *oscillate* with random period between two extreme values in the subregion $4 < G_f R_f^{-2} < 40$. The envelopes of these extreme values are indicated by the solid lines which enclose the crosshatched cell-shaped areas in Figs. 2 and 3. For vertically upward flow such oscillation was not observed; nevertheless, the data for $2 < G_f R_f^{-2} < 40$ in Figs. 4 and 5 delineate a curve similar in shape to the curve which passes through the lower values of Δt in Figs. 2 and 3. A hypothesis to explain the oscillatory behavior described in the preceding is presented below in conjunction with the correlation of the data.

A second anomaly was encountered under experimental conditions corresponding to vertically downward flow and $G_f R_f^{-2} > 4$. Under

* In the case of set one these curves are extensions of the curves which appear in [11] as shown by way of example, in Fig. 1.

Table 1. Experimental data obtained using test cylinder with D = 0·4511 *in. and horizontal forced flow* (θ = 90 *deg*)

$G_f R_f^{-2}$	q (Btu h^{-1} ft^{-2})	t_b = 50 deg F		t_b = 70 deg F		t_b = 100 deg F		t_b = 140 deg F	
		U(ft s^{-1})	Δt(deg F)	U(ft s^{-1})	Δt(deg F)	U(ft s^{-1})	Δt(deg F)	U(ft s^{-1})	Δt(deg F)
0·125	10302	0·1925	34·5	0·2225	30·8	0·2457	26·6	0·2660	23·4
0·125	13941	0·2248	42·3	0·2544	38·0	0·2772	32·8	0·3000	29·2
0·125	17457	0·2537	49·3	0·2824	44·6	0·3060	38·8	0·3286	34·6
0·125	20943	0·2800	55·9	0·3082	50·8	0·3313	44·5	0·3572	40·3
0·125	27826	0·3272	68·0	0·3535	62·3	0·3782	55·5	0·4045	50·3
0·125	35048	0·3702	79·4	0·3943	72·9	0·4193	65·7		
0·125	44019	0·4194	92·8						
0·25	10302	0·1486	38·7	0·1695	34·5	0·1875	30·3	0·2028	27·0
0·25	13941	0·1730	47·0	0·1934	42·3	0·2115	37·3	0·2302	34·0
0·25	17457	0·1946	54·7	0·2152	49·8	0·2333	44·1	0·2525	40·3
0·25	20943	0·2143	61·8	0·2352	56·9	0·2526	50·5	0·2720	46·0
0·25	27826	0·2500	75·0	0·2691	69·3	0·2867	62·3	0·3074	57·1
0·25	35048	0·2830	87·5	0·2994	80·8	0·3178	73·4		
0·25	44019	0·3194	101·8	0·3370	95·5				
0·25	52471	0·3497	114·2	0·3691	108·5				
0·25	60988	0·3772	125·5	0·3995	121·2				
0·25	69454	0·4044	136·8	0·4288	133·6				
0·50	10302	0·1147	43·0	0·1285	38·5	0·1422	34·2	0·1533	30·5
0·50	13941	0·1337	52·2	0·1468	47·3	0·1607	42·2	0·1757	39·0
0·50	17457	0·1498	60·3	0·1637	55·5	0·1775	49·8	0·1932	46·3
0·50	20943	0·1653	68·2	0·1788	63·3	0·1917	56·8	0·2082	52·8
0·50	27826	0·1928	82·7	0·2046	76·9	0·2176	69·9	0·2335	64·6
0·50	35048	0·2177	96·5	0·2284	90·1	0·2413	82·3		
0·50	44019	0·2465	112·8	0·2555	105·5				
0·50	52471	0·2692	126·0	0·2798	119·8				
0·50	60988	0·2892	138·1	0·3034	133·7				
0·50	69454	0·3083	149·8	0·3260	147·5				
0·50	77045			0·3455	159·7				
1·00	10302	0·0877	47·5	0·0970	42·7	0·1069	38·1	0·1154	34·2
1·00	13941	0·1023	57·8	0·1112	52·3	0·1218	47·5	0·1323	43·8
1·00	17457	0·1143	66·7	0·1242	61·7	0·1345	55·9	0·1462	52·2
1·00	20943	0·1261	75·3	0·1358	70·3	0·1453	63·5	0·1578	59·7
1·00	27826	0·1473	91·5	0·1553	85·2	0·1647	77·8	0·1777	73·3
1·00	35048	0·1667	106·8	0·1740	100·0	0·1830	91·8		
1·00	44019			0·1948	116·9				
1·00	52471			0·2133	132·7				
1·00	60988			0·2306	147·8				
1·00	69454			0·2477	162·7				
2·00	10302	0·0667	52·4	0·0727	46·6	0·0800	42·0	0·0861	37·8
2·00	13941	0·0780	64·2	0·0838	57·5	0·0917	52·7	0·0992	48·4
2·00	17457	0·0875	74·1	0·0936	67·5	0·1013	62·0	0·1095	57·5
2·00	20943	0·0957	82·8	0·1023	77·1	0·1097	70·7	0·1182	66·0
2·00	27826	0·1116	100·3	0·1172	93·5	0·1237	85·7		
2·00	35048	0·1267	117·5	0·1313	109·7	0·1379	101·5		
2·00	44019			0·1474	128·6				
2·00	52471			0·1626	147·1				
2·00	60988			0·1756	163·3				

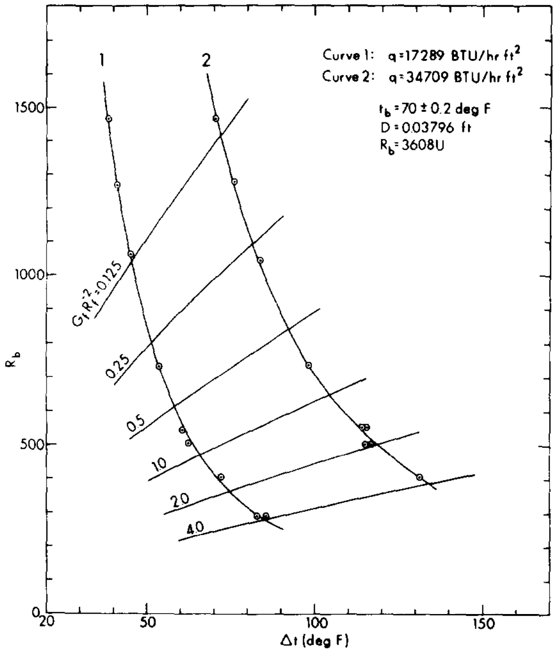

FIG. 6. Experimental data obtained using test cylinder with $D = 0{\cdot}4555$ in., $t_b = 70$ deg F and vertically downward forced flow ($\theta = 180$ deg).

Table 2. Experimental data obtained using test cylinder with D = 0·4555 *in.*

$G_f R_f^{-2}$	q (Btu h^{-1} ft^{-2})	$\theta = 0$ deg, $t_b = 70$ deg F		$\theta = 0$ deg, $t_b = 100$ deg F		$\theta = 90$ deg, $t_b = 70$ deg F		$\theta = 90$ deg, $t_b = 100$ deg F		$\theta = 180$ deg, $t_b = 70$ deg F	
		R_b	Δt (deg F)	R_b	Δt (deg F)	R_b	Δt (deg F)	R_b	Δt (deg F)	R_b	Δt (deg F)
0·125	17289	1031	45·0	1589	39·1	1034	45·3	1589	39·1	1038	45·5
0·125	34709	1417	72·1	2155	64·9	1426	72·8	2165	65·4	1417	72·1
0·25	17289	781	50·0	1198	43·7	791	50·9	1199	43·8	798	51·7
0·25	34709	1076	79·8	1623	71·8	1088	81·3	1637	72·8	1092	81·5
0·5	17289	595	55·8	909	49·1	602	56·9	914	49·5	613	58·3
0·5	34709	818	88·5	1228	80·0	828	90·0	1242	81·5	842	92·2
1·0	17289	454	62·4	689	55·1	457	63·0	695	55·8	472	65·9
1·0	34709	622	98·0	934	89·8	629	99·8	948	91·8	650	104·3
2·0	17289	348	70·4	523	61·7	344	69·0	528	62·7	363	74·7
2·0	34709	474	108·9	710	100·6	476	109·7	718	102·4	504	118·9
4·0*	17289					256	74·4	395	68·9	282	85·1
4·0*	34709							539	111·9	388	133·8

* These data points belong, strictly speaking, to zone three (defined by $2 < G_f R_f^{-2} < 40$); nevertheless, as may be seen in Figs. 2, 3 and 6, these points follow the trend established in zones one and two (defined by $G_f R_f^{-2} < 2$).

these conditions the surface temperature of the test specimen fluctuated erratically to such an extent that no coherent data could be collected—an explanation for this behavior is presented in the section which follows.

CORRELATION OF DATA AND DISCUSSION OF RESULTS

In [11] two different methods of accounting for the influence of property variation on forced convection heat transfer to water have been investigated. In the first of these methods all properties are evaluated at the mean film temperature, and in the second method the pertinent dimensionless parameters (N, R and P) are corrected for property variation by means of individual correction factors in the following form:

$$Q_{scq} = Q_s \left(\frac{Q_b}{Q_s}\right)^q. \tag{14}$$

In equation (14) the symbol Q stands for any dimensionless parameter and q, called the correction factor exponent, is a number between zero and one. The symbol Q_{scq} is to be read "surface Q with correction factor exponent q." [11] provides the following two correlation equations for forced convection heat transfer from cylinders to liquids in crossflow corresponding to the two aforementioned methods of accounting for property variation:

$$N_{f,F} = (0{\cdot}255 + 0{\cdot}699 R_f^{0{\cdot}50}) P_f^{0{\cdot}29}, \tag{15}$$

$$N_{scn=0{\cdot}50,F} = (0{\cdot}221 + 0{\cdot}693 R_{scr=0{\cdot}75}^{0{\cdot}50}) P_b^{0{\cdot}29}. \tag{16}$$

Although the errors incurred by using equation (16) are less than by equation (15), the difference between the two is not great, and hence both avenues of approach will be explored here.

If equation (15) is plotted as shown in Figs. 4 and 5, it is seen that the values for Δt predicted by the equation become progressively higher than the experimental results as $G_f R_f^{-2}$ increases. This divergence reflects the existence of higher actual heat transfer coefficients than predicted by equation (15)—the divergence is ascribable to the superposition in the actual experiments of natural convection effects not accounted for by the equation. In Figs. 4 and 5 the discrepancy between the experimentally measured heat transfer coefficients and the corresponding values calculated from equation (15) is approximately 5 per cent when $G_f R_f^{-2} = 0{\cdot}5$ and 10 per cent when $G_f R_f^{-2} = 2$. Since an error of 5 per cent in heat transfer calculations is usually considered to be acceptable, whereas an error as high as 10 per cent may not, the values $G_f R_f^{-2} = 0{\cdot}5$ and $G_f R_f^{-2} = 2$ are taken here to define two zones—zone one defined by $G_f R_f^{-2} < 0{\cdot}5$; zone two defined by $0{\cdot}5 < G_f R_f^{-2} < 2$—in the first of which equation (15) may be used with negligible error, and in the second of which a correlation equation that reflects the combined effects of natural and forced convection must be used.

In order to determine the required correlation equation for zone two, it was assumed that the Nusselt number is a function of GR^{-2} and that natural convection can be accounted for by simply adding a term to the forced convection equations in the following manner:

$$N_f = \left[0{\cdot}255 + 0{\cdot}699 R_f^{0{\cdot}50} + C\left(\frac{G_f}{R_f^2}\right)^m \times G_f^{0{\cdot}25}\right] P_f^{0{\cdot}29}, \tag{17}$$

$$N_{scn=0{\cdot}50} = \left[0{\cdot}221 + 0{\cdot}693 R_{scr=0{\cdot}75}^{0{\cdot}50} + K \times \left(\frac{G_{scg}}{R_{scr=0{\cdot}75}^2}\right)^n G_{scg}^{0{\cdot}25}\right] P_b^{0{\cdot}29}. \tag{18}$$

C, K, m, n and g are constants whose values were determined to be those shown in Table 3 based

Table 3. Empirically determined constants for use in equations (17) *and* (18)

θ (deg)	C	K	m, n	g
0	0·0330	0·0224	0·30	0·50
90	0·0188	0·0131	1·00	0·50
180	−0·0240	−0·0246	0·15	0·50

on the experimental data listed in Tables 1 and 2. The choice of the form of the additive natural convection terms in equations (17) and (18) is based upon two assumptions: (1) that the natural convection contribution to the combined Nusselt number is a *fraction* of the pure natural convection Nusselt number which would prevail for the same Δt; and (2) that this fraction is proportional to the quantity GR^{-2} raised to a power. Since, in the range of Grashof numbers encountered here ($5 \times 10^4 < G_f < 4 \times 10^6$), the Nusselt number for pure natural convection is proportional to the product of $G^{0\cdot25}P^x$ where x lies somewhere between $\frac{1}{4}$ and $\frac{1}{3}$, the form chosen for the additive term follows directly from the two assumptions stated.

Attempts were made to correlate the data in zone two by various schemes involving the vectorial addition of natural and forced convection effects. All these attempts failed, and this failure is attributed to the presence in zone two of separated flow, which is a complex nonlinear flow phenomenon.

The errors incurred by using equations (17) and (18) with the values of C, K, m, n and g listed in Table 3 are indicated in Table 4. The errors for equations (15) and (16) are included in Table 4 for purposes of comparison with the errors for equations (17) and (18). The mean deviations between the calculated and experimental values of the Nusselt number listed in Table 4 for equations (17) and (18) are all less than 3 per cent, which level of error is considered to be acceptable. Equation (17) is plotted in Figs. 4 and 5 for purposes of visual comparison with the experimental data. Obviously, equation (17) approaches equation (15) for increasing values of R.

The data which lie in the region designated here as zone three, defined by $2 < G_fR_f^{-2} < 40$, exhibit the anomalous behavior described under "Experimental Procedure and Data." Zone three will be considered in detail following a discussion of the remaining region, designated as zone four and defined by $G_fR_f^{-2} > 40$.

It is postulated that the Grashof number in zone four is so large in comparison with the forced flow Reynolds number that natural convection effects predominate; that is, in zone four the boundary layer flow around the entire circumference of the heated cylinder is laminar and separation does not occur. Since the boundary layer around a cylinder normally separates when the crossflow Reynolds number exceeds approximately 10, but apparently does not do so in zone four for Reynolds numbers considerably higher than this figure, it can be inferred that the superposition of natural convection serves to *stabilize*

Table 4. Errors incurred by various correlation equations relative to the data in Tables 1 and 2

θ (deg)	Equation No.	$0{\cdot}125 \leqslant G_fR_f^{-2} \leqslant 0{\cdot}5$				$0{\cdot}5 < G_fR_f^{-2} \leqslant 4$			
		E_{av} (%)	σ (%)	E_m (%)	E_{max} (%)	E_{av} (%)	σ (%)	E_m (%)	E_{max} (%)
0	15	−2·4	2·4	2·8	6·4	−6·2	2·2	6·2	8·5
	16	−2·0	2·4	2·5	6·1	−5·8	2·3	5·8	8·2
	17	−0·2	2·0	1·6	3·6	−0·9	2·3	2·1	3·6
	18	−0·3	2·1	1·7	3·7	−1·8	2·3	2·6	3·7
90	15	−1·0	1·9	1·8	5·0	−5·6	2·7	5·6	11·5
	16	−0·7	1·8	1·6	4·3	−5·3	2·5	5·3	10·9
	17	−0·4	1·8	1·6	5·0	−1·0	2·3	2·2	4·9
	18	−0·2	1·7	1·4	4·7	−1·1	2·3	2·2	4·7
180	15	2·1	2·3	2·3	4·3	4·9	1·9	4·9	7·8
	16	2·6	2·3	2·7	4·9	5·3	2·0	5·3	8·4
	17	0	2·2	2·0	2·3	0	1·3	1·2	1·6
	18	0·3	2·2	2·0	2·6	−0·3	1·7	1·5	2·5

the flow in zone four. On the basis of this observation, it is possible to devise a simple semi-empirical theory whereby the heat transfer coefficient can be calculated in zone four with reasonable accuracy.

The theory begins by considering the natural convection field of flow near a heated vertical flat plate and the associated heat transfer which, it is known, may be expressed as follows:

$$N_{N,x} = f_1(G_x, P), \tag{19}$$

where x is the distance from the leading edge. Suppose now that there exists a hypothetical fluid all of whose properties are identical to a particular real fluid except that the coefficient of volumetric expansion of the hypothetical fluid is equal to zero. In such a fluid buoyant force could not exist and natural convection would not occur. However, such a fluid could be *forced* to flow upward parallel to the surface of a heated vertical flat plate, and the resulting heat transfer could be described by a forced convection equation of the following form:

$$N_{F,x} = f_2(R_{F,x}, P). \tag{20}$$

Conceptually, the velocity of forced flow can be adjusted to render the local rate of heat transfer by forced convection at the distance x from the leading edge of the plate in the presence of the hypothetical fluid equal to that obtained in natural convection when the plate is surrounded by the real fluid. This conceptual experiment describes what shall be referred to hereafter as "heat transfer equivalence" between natural and forced convection relative to the point x. The concept of heat transfer equivalence, which can be readily extended to apply to an area, forms the first of two fundamental ideas that underlie the semi-empirical theory developed herein to account for combined natural and forced convection effects.

The second fundamental idea upon which the theory rests is the assumption that combined natural and forced convection effects can be accounted for by means of the vectorial sum of the equivalent and forced convection Reynolds numbers. The procedure will be demonstrated by applying it to the geometry studied in the present work, namely, that of a heated horizontal cylinder exposed to crossflow.

It is well known that the Nusselt number for heat transfer by forced convection from bodies to *external laminar flows* is nearly proportional to the product $R_{f,F}^{1/2} P_f^{1/3}$; thus

$$N_{f,F} = C_F R_{f,F}^{1/2} P_f^{1/3}. \tag{21}$$

For natural convection within the range of Grashof numbers studied here ($5 \times 10^4 < G_f < 4 \times 10^6$) the analogous formula for a cylinder is

$$N_{f,N} = C_N G_f^{1/4} P_f^{1/3}. \tag{22}$$

The "equivalent Reynolds number" in the sense defined earlier may now be obtained by setting $N_{f,F}$ equal to $N_{f,N}$; thus

$$C_F R_{f,F,\mathrm{eq}}^{1/2} P_f^{1/3} = C_N G_f^{1/4} P_f^{1/3}$$

or

$$R_{f,F,\mathrm{eq}} = (C_N^2/C_F^2)\, G_f^{1/2} = (C_{\mathrm{eq}} G_f)^{1/2} \tag{23}$$

where $C_{\mathrm{eq}} = (C_N^4/C_F^4)$. If, now, a *horizontal* forced flow ($\theta = 90$ deg) is superimposed upon the equivalent *vertical* flow the magnitude of the resultant Reynolds number, designated $|R_f|$, obtained by vector addition is

$$|R_f| = (R_{f,F,\mathrm{eq}}^2 + R_{f,F}^2)^{1/2} = (C_{\mathrm{eq}} G_f + R_{f,F}^2)^{1/2}. \tag{24}$$

The method of correlation proposed here assumes that if equation (21) is modified by replacing the Reynolds number that appears therein by $|R_f|$, then the modified equation yields the Nusselt number for combined natural and forced convection for this geometry:

$$N_{f,\theta=90} = C_F (C_{\mathrm{eq}} G_f + R_{f,F}^2)^{1/4}\, P_f^{1/3}. \tag{25}$$

If the direction of the superimposed forced flow is vertical ($\theta = 0$ deg), the analogous equation is:

$$N_{f,\theta=0} = C_F [(C_{\mathrm{eq}} G_f)^{1/2} + R_{f,F}]^{1/2}\, P_f^{1/3}. \tag{26}$$

If the constants C_N, C_F and C_{eq} are taken equal to 0·464, 0·406 and 1·70, respectively, then equations (25) and (26) plot as indicated in Fig. 7. The agreement between the equations and the experimental data included in Fig. 7 is considered to be sufficiently good to conclude that the theory presented above predicts the *trends* of the data reasonably well. However, it is

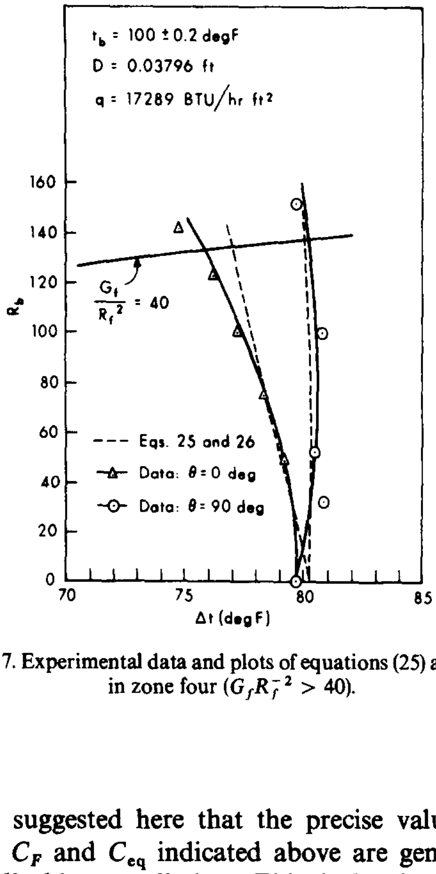

FIG. 7. Experimental data and plots of equations (25) and (26) in zone four ($G_f R_f^{-2} > 40$).

not suggested here that the precise values of C_N, C_F and C_{eq} indicated above are generally applicable to cylinders. This lack of general applicability stems from the fact that the experimental results obtained in the present investigation, for those tests in which natural convection predominated, relate to a *particular confined space* (duct) in which these experiments were conducted.

The discussion now reverts to consideration of zone three, defined by $2 < G_f R_f^{-2} < 40$. It is now apparent that this zone lies between a region in which forced convection predominates (zone two) and a region in which natural convection predominates (zone four). This circumstance leads to the hypothesis that the oscillation of the cylinder surface temperature between two extreme values described previously is the result of periodic changes in the boundary layer flow—the lower extreme value is believed to correspond to the periodic establishment of laminar boundary layer flow resembling the flow associated with pure natural convection, and the higher extreme value is believed to correspond to the periodic establishment of the kind of separated boundary layer flow normally encountered near cylinders in forced crossflow.

It was mentioned under "Experimental Procedure and Data" that the surface temperature of the cylinder behaved so erratically for vertically downward forced flow for $G_f R_f^{-2} > 4$ as to preclude the taking of meaningful data. The reasons for this erratic behavior will now be given. The difficulty encountered here was fundamental; because, for vertically downward forced flow and $G_f R_f^{-2} > 4$, the experiments attempted were not well-defined nor did they reach steady state. That this was so becomes obvious if one remembers that, for $G_f R_f^{-2} > 4$, natural convection effects are approximately equal to or are greater than forced convection effects. Since in all cases now under discussion *the net flow was vertically downward*, it follows that the heated fluid that rose in the natural convection plume above the cylinder *must have eventually descended* and this descending heated fluid caused the surface temperature of the cylinder to behave erratically. The process described is one of *recirculation* in which the pattern of flow is three-dimensional and depends upon the geometry of the duct in which the experiments were performed; furthermore, since such recirculation resulted in the continuous "reheating" of some fluid, steady state was never reached in these particular experiments. It is anticipated that the problems described here will be encountered in any study of combined natural and forced convection heat transfer when the direction of the forced flow in vertically downward and $G_f R_f^{-2} > 4$.

RESUME AND CONCLUSIONS

It has been demonstrated that the data for convective heat transfer from horizontal cylinders to water in crossflow gathered in the course of this study can be relegated to four zones depending upon the value of the ratio $G_f R_f^{-2}$. In the first zone, defined by $G_f R_f^{-2} < 0.5$, the predominant heat transfer mechanism is forced convection and previously published forced convection equations may be applied with negligible error (less about 5 per cent). In the second zone, defined by $0.5 < G_f R_f^{-2} < 2$, the predominant heat transfer mechanism is still forced convection, but here natural convection affects the overall heat transfer coefficient by as much as 10 per cent and this is not considered to be negligible. Empirical correlation equations, (17) and (18), have been developed by means of which it is possible to account for natural convection in zone two for three directions of forced flow: vertically upward, horizontal, and vertically downward. Equations (17) and (18) may be used in zone one to obtain a slightly better estimate for the Nusselt number than is provided by equations (15) and (16). The new correlation equations have been constructed on the basis of two assumptions: (1) that the natural convection contribution to the combined Nusselt number is a *fraction* of the pure natural convection Nusselt number which would prevail for the same Δt; and (2) that this fraction is proportional to the quantity GR^{-2} raised to a power.

Attempts were made to correlate the data in zone two by various schemes involving the vectorial addition of natural and forced convection effects. All these attempts failed, and this failure is attributed to the presence in zone two of separated flow, which is a complex nonlinear flow phenomenon.

Zone three, defined by $2 < G_f R_f^{-2} < 40$, is a region in which natural and forced convection effects are of the same order of magnitude. In a subregion of this zone ($4 < G_f R_f^{-2} < 40$) the heat transfer process is unsteady when the direction of the forced flow is horizontal; under such circumstances the heat transfer coefficient oscillates with random period between two extreme values. It is hypothesized that this oscillation is a result of periodic changes in the boundary layer flow—the lower extreme value of the heat transfer coefficient is believed to correspond to the periodic establishment of laminar boundary flow resembling the flow associated with pure natural convection, and the higher extreme value is believed to correspond to the periodic establishment of the kind of separated boundary layer flow normally encountered near cylinders.

Zone four, defined by $G_f R_f^{-2} > 40$, is a region in which natural convection effects predominate. A simple theory has been developed to describe the behavior of the heat transfer coefficient in zone four. This theory is applicable only to laminar boundary layer flow, i.e. when neither separation nor turbulence occurs.

The data for horizontal forced flow in zone four plotted in Figs. 2 and 3 indicate that the heat transfer coefficient goes through a shallow minimum before reaching its free convection value at zero Reynolds number. This curious phenomenon has been observed by Collis and Williams [12] among others.

The information obtained in this study, in conjunction with the low Reynolds number data and the correlations presented by Hatton *et al.* [7], leads to the conclusion that a fifth zone exists which could not be investigated with the experimental apparatus used in the present study. This zone lies close to the origin in such plots as Fig. 1, and is indicated graphically in Fig. 8. Zone five is triangular in shape. One of its boundaries is the vertical axis, $\Delta t = 0$; another is the line $G_f R_f^{-2} = 40$; and the third boundary, which is described here in general terms due to a lack of precise quantitative data, is a line that represents the locus of the *maximum crossflow Reynolds number* for laminar (unseparated) boundary layer flow around a cylinder in the presence of natural convection. The conditions corresponding to zone five can be achieved in the laboratory only by using cylinders having very small diameters

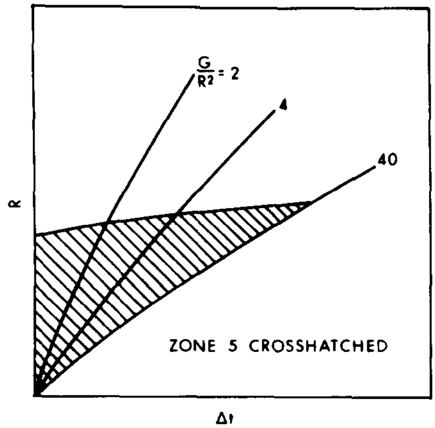

FIG. 8. Location of zone five.

(fine wires). Since the flow in zone five is laminar, the necessary conditions for applying the concept of heat transfer equivalence and vectorial addition to account for combined natural and forced convection are satisfied. The data gathered by Hatton *et al.* [7] lay in the range $0{\cdot}01 < R_f < 45$ and it is postulated here that the reason their vectorial addition correlation technique succeeded to the extent that it did was because in most, if not all, their experiments the boundary layer flow was laminar.

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REFERENCES

1. F. KREITH, *Principles of Heat Transfer*, Second Ed. International Textbook, Scranton, Pennsylvania (1966).
2. E. M. SPARROW and J. L. GREGG, Buoyancy effects in forced convection flow and heat transfer, *J. Appl. Mech.* **81**, 133–134 (1959).
3. G. K. SHARMA and S. P. SUKHATME, Combined free and forced convection heat transfer from a heated tube to a transverse air stream, *J. Heat Transfer* **91**, 457–459 (1969).
4. P. H. OOSTHUIZEN and S. MADAN, Combined convective heat transfer from horizontal cylinders in air, *J. Heat Transfer* **92**, 194–196 (1970).
5. P. H. OOSTHUIZEN and S. MADAN, The effect of flow direction on combined convective heat transfer from cylinders to air, *J. Heat Transfer* **93**, 240–242 (1971).
6. B. G. VAN DER HEGGE ZIJNEN, Modified correlation formulae for the heat transfer by natural and by forced convection from horizontal cylinders, *Appl. Sci. Res.* **A6**, 129–140 (1956).
7. A. P. HATTON, D. D. JAMES and H. W. SWIRE, Combined forced and natural convection with low-speed air flow over horizontal cylinders, *J. Fluid Mech.* **42**, Part 1, 17–31 (1970).
8. T. W. JACKSON and H. H. YEN, Combining forced and free convection equations to represent combined heat-transfer coefficients for a horizontal cylinder, *J. Heat Transfer* **93**, 247–248 (1971).
9. B. GEBHART and L. PERA, Mixed convection from long horizontal cylinders, *J. Fluid Mech.* **45**, Part 1, 49–64 (1971).
10. R. M. FAND, Acoustic water tunnel, *J. Acoust. Soc. Am.* **38**, 561–569 (1965).
11. R. M. FAND and K. K. KESWANI, The influence of property variation on forced convection heat transfer to liquids, *Int. J. Heat Mass Transfer* **15**, 1515–1536 (1972).
12. D. C. COLLIS and M. J. WILLIAMS, Two dimensional convection from heated wires at low Reynolds numbers, *J. Fluid Mech.* **6**, 357–384 (1959).

TRANSFERT THERMIQUE PAR CONVECTION MIXTE NATURELLE ET FORCEE DE CYLINDRES HORIZONTAUX DANS L'EAU

Résumé—On a obtenu des résultats expérimentaux pour le transfert thermique par convection depuis des cylindres horizontaux (diamètre approximatif de 1,27 cm) dans l'eau en écoulement frontal. On montre que ces résultats peuvent être reliés à quatre zones, dépendant de la valeur du rapport $G_f R_f^{-2}$, où G_f et R_f sont respectivement les nombres de Grashof et de Reynolds avec propriétés évaluées à la température moyenne du film. Dans la première zone définie par $G_f R_f^{-2} < 0{,}5$ le mécanisme prédominant du transfert thermique est la convection forcée et les équations de convection forcée déjà publiées peuvent être appliquées avec une erreur négligeable (moins de 5 pour cent). Dans la seconde zone où $0{,}5 < G_f R_f^{-2} < 2$ le mécanisme principal de transfert thermique est toujours la convection forcée mais ici la convection naturelle peut affecter de 10 pour cent le coefficient global de transfert thermique ce qui n'est pas négligeable. On a présenté des équations empiriques au moyen desquelles il est possible de tenir compte de la convection naturelle pour trois directions de l'écoulement forcé: verticalement vers le haut, horizontal, vericalement vers la bas. La troisième zone définie par $2 < G_f R_f^{-2} < 40$ est une région dans laquelle les effets des convections naturelle et forcée sont du même ordre de grandeur. Dans une sous-région de cette zone ($4 < G_f R_f^{-2} < 40$) le processus de transfert thermique n'est pas stable quand la direction de l'écoulement forcé est horizontale; sous ces conditions le coefficient de transfert thermique oscille avec une période due au hasard entre deux valeurs extrêmes. On a émis l'hypothèse que cette oscillation résulte de changements périodiques dans l'écoulement de couche limite. On pense que la valeur extrême la plus basse du coefficient de transfert thermique correspond à l'établissement périodique de l'écoulement de couche limite ressemblant à l'écoulement associé à la convection naturelle pure, et que la valeur extrême la plus grande correspond à l'établissement périodique de l'écoulement de couche limite séparée normalement rencontré près des cylindres dans l'écoulement frontal forcé. La quatrième zone définie par $G_f R_f^{-2} > 40$ est une région dans laquelle prédominent les effets de convection naturelle. On a développé une théorie simple pour décrire le comportement du coefficient du transfert thermique dans cette zone. On a identifié une cinquième zone qui ne contient aucun des résultats rassemblés ici.

KOMBINIERTE FREIE UND ERZWUNGENE KONVEKTIVE WÄRMEÜBERTRAGUNG VON HORIZONTALEN ZYLINDERN AN WASSER

Zusammenfassung—Es wurden experimentelle Daten für konvectiven Wärmetransport von horizontalen Zylindern (Durchmesser etwa 12 mm) an Wasser im Kreuzstrom erhalten. Es wird gezeigt, dass diese Ergebnisse vier Zonen zugeteilt werden können, abhängig vom Wert des Verhältnisses $G_f \cdot R_f^{-2}$, wobei G_f und R_f die Grashof- und Reynolds-Zahlen sind, die mit Werten der mittleren Filmtemperatur gebildet wurden. In der ersten Zone $G_f R_f^{-2} < 0{,}5$ erfolgt Wärmetransport vorwiegend durch erzwungene Konvektion, und früher veröffentlichte Gleichungen für erzwungene Konvektion können mit vernachlässigbarem Fehler (weniger als 5%) angewendet werden. Auch in der zweiten Zone $0{,}5 < G_f R_f^{-2}$ erfolgt der Wärmetransport vorwiegend durch erzwungene Konvektion, aber hier beträgt die freie Konvektion ungefähr 10% des gesamten Wärmetransports, was nicht mehr zu vernachlässigen ist. Empirische Korrelationsgleichungen sind angegeben, um die freie Konvektion in Zone zwei für drei Richtungen der erzwungenen Strömung zu berücksichtigen, nämlich vertikal nach oben, horizontal und vertikal nach unten. Zone drei, definiert durch $2 < G_f \cdot R_f^{-2} < 40$, ist ein Bereich, in welchem die Effekte der freien und erzwungenen Konvektion von gleicher Größenordnung sind. In einem Unterbereich, $4 < G_f R_f^{-2} < 40$ ist der Wärmeübertragungsprozeß unstetig, bei horizontaler, erzwungener Strömung, dabei schwankt der Wärmetransportkoeffizient mit zufälliger Periode zwischen zwei Extremwerten. Es wird angenommen, dass diese Schwankungen das Ergebnis periodischer Änderungen in der Grenzschichtströmung sind: das Minimum des Wärmetransportkoeffizienten soll dem periodischen Auftreten von laminarer Grenzschichtströmung entsprechen wie sie bei reiner freier Konvektion herrscht und das Maximum soll vom periodischen Auftreten von abgelöster Grenzschichtströmung kommen, die gewöhnlich nahe Zylindern bei Anströmung auftritt. Zone vier $G_f R_f^{-2} > 40$ ist ein Bereich, in welchem die freie Konvektion vorherrscht. Es wurde eine einfache Theorie entwickelt, um das Verhalten des Wärmetransportkoeffizienten in Zone 4 zu beschreiben. Eine fünfte Zone wurde identifiziert, welche keine der hier erhaltenen Daten enthält.

ТЕПЛООБМЕН ГОРИЗОНТАЛЬНЫХ ЦИЛИНДРОВ С ВОДОЙ ПРИ СОВМЕСТНОЙ ЕСТЕСТВЕННОЙ И ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация—Получены экспериментальные данные по конвективному теплообмену при поперечном обтекании водой горизонтальных цилиндров диаметром приблизительно ½ дюйма. Показано, что эти данные могут быть применены к четырем зонам в зависимости от величины отношения $G_f R_f^{-2}$, где G_f и R_f есть соответственно числа Грасгофа и Рейнольдса. В этом случае характеристики определялись при средней температуре пленки. В перввй зоне, определяемой неравенством $G_f R_f^{-2} < 0{,}5$, доминирующим механизмом теплообмена является вынужденная конвекцтя, и здесь с пренебрежимо малой погрешностью (не менее 5%) могут быть использованы ранее полученные уравенения для вынужденной конвекции. Во второй зоне, определяемой неравенством $0{,}5 < G_f R_f^{-2} < 2$, доминирующим механизмом теплообмена всё ещё остается вынужденная конвекция, но в этом случае на общий коэффициент теплообмена (в пределах 10%) влияет естественная конвекция, и ею нельзя пренебьегать. Приведены эмпирические уравнения, с помощью которых можно учитывать естественную конвекцию во второй зоне для трех направлений вынужденного потока: вертикального вверх, горизонтального и вертикального вниз. Третья зона, определяемая неравенством $2 < G_f R_f^{-2} < 40$, представляет собой область, в которой естественная и вынужденная конвекция имеют один и тот же порядок величины. В подобласти этой зоны ($4 < G_f R_f^{-2} < 40$) в случае горизонтального направления вынужденного потока процесс теплообмена является нестационарным. В таких условиях коэффициент теплообмена колеблется со случайной частотой между двумя предельными значениями. Предполагается, что изменение есть результат периодических изменений течения пограничного слоя. При этом нижнее предельное значение коэффициента теплообмена соответствует периодическому установлению ламинарного течения в пограничном слое, напоминающему течение при естественной конвекции, а верхнее предельное значение-периодическому установлению такого отрывного течения в нограничном слое, которое встречается вблизи цилиндров в поперечном потоке при вынужденной конвекции. Четвертая зона, определяемая неравенством $G_f R_f^{-2} > 40$, представляет собой зону, в которой преобладают эффекты естественной конвекции. Развита простая теория для описания поведения коэффициента теплообмена в четвертой зоне. Определена пятая зона, для которой не найдено значения отношения $G_f R_f^{-2}$.